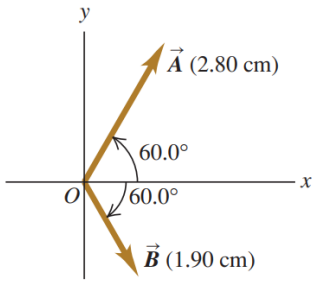
Exercise 1.39

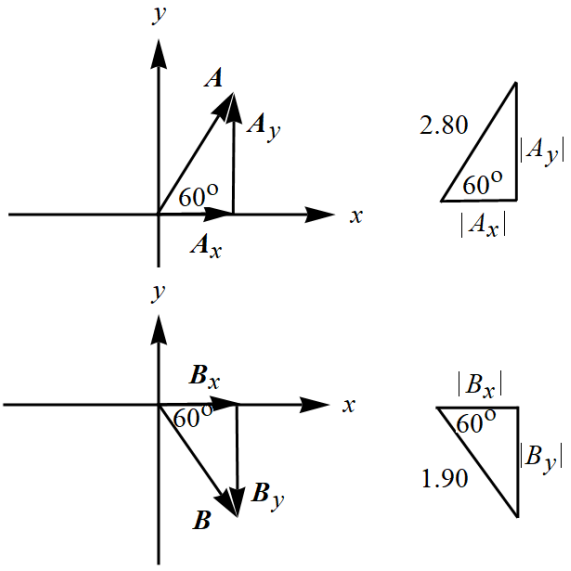
Vector $\vec{A}$ is 2.80 cm long and is 60.0° above the x-axis in the first quadrant. Vector $\vec{B}$ is 1.90 cm long and is 60.0° below the x-axis in the fourth quadrant (Fig. E1.39). Use components to find the magnitude and direction of (a) $\vec{A} + \vec{B}$; (b) $\vec{A} - \vec{B}$; (c) $\vec{B} - \vec{A}$. In each case, sketch the vector addition or subtraction and show that your numerical answers are in qualitative agreement with your sketch.

Figure **E1.39**



Solution

Decompose each of the given vectors into components along the x- and y-axes. Then draw the resulting triangles consisting of the magnitudes.



Use trigonometry to determine the unknown components in each triangle.

$$\cos 60° = \frac{|A_x|}{2.80} \quad \rightarrow \quad |A_x| = 2.80 \cos 60° \qquad \cos 60° = \frac{|B_x|}{1.90} \quad \rightarrow \quad |B_x| = 1.90 \cos 60°$$

$$\sin 60° = \frac{|A_y|}{2.80} \quad \rightarrow \quad |A_y| = 2.80 \sin 60° \qquad \sin 60° = \frac{|B_y|}{1.90} \quad \rightarrow \quad |B_y| = 1.90 \sin 60°$$

All of the vector components point in the positive x- and y-directions except for $\mathbf{B}_y$, which points

in the negative y-direction. B_y therefore has a minus sign.

$$\begin{aligned} A_x &= 2.80\cos 60^\circ \\ A_y &= 2.80\sin 60^\circ \\ B_x &= 1.90\cos 60^\circ \\ B_y &= -1.90\sin 60^\circ \end{aligned}$$

Write each of the vectors in terms of the components.

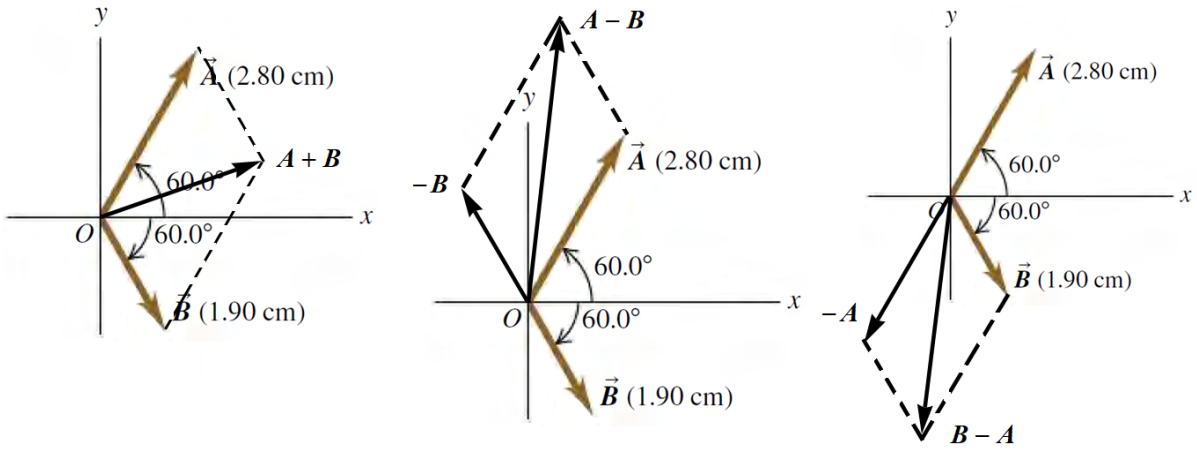
$$\begin{aligned} \mathbf{A} &= \langle A_x, A_y\rangle = \langle 2.80\cos 60^\circ, 2.80\sin 60^\circ\rangle \text{ cm} \\ \mathbf{B} &= \langle B_x, B_y\rangle = \langle 1.90\cos 60^\circ, -1.90\sin 60^\circ\rangle \text{ cm} \end{aligned}$$

Now determine the desired sum and differences.

$$\begin{aligned} \mathbf{A}+\mathbf{B} &= \langle 2.80\cos 60^\circ, 2.80\sin 60^\circ\rangle \text{ cm} + \langle 1.90\cos 60^\circ, -1.90\sin 60^\circ\rangle \text{ cm} \\ &= \langle 2.80\cos 60^\circ + 1.90\cos 60^\circ, 2.80\sin 60^\circ - 1.90\sin 60^\circ\rangle \text{ cm} \\ &\approx \langle 2.35, 0.779\rangle \text{ cm} \end{aligned}$$

$$\begin{aligned} \mathbf{A}-\mathbf{B} &= \langle 2.80\cos 60^\circ, 2.80\sin 60^\circ\rangle \text{ cm} - \langle 1.90\cos 60^\circ, -1.90\sin 60^\circ\rangle \text{ cm} \\ &= \langle 2.80\cos 60^\circ - 1.90\cos 60^\circ, 2.80\sin 60^\circ + 1.90\sin 60^\circ\rangle \text{ cm} \\ &\approx \langle 0.45, 4.07\rangle \text{ cm} \end{aligned}$$

$$\begin{aligned} \mathbf{B}-\mathbf{A} &= \langle 1.90\cos 60^\circ, -1.90\sin 60^\circ\rangle \text{ cm} - \langle 2.80\cos 60^\circ, 2.80\sin 60^\circ\rangle \text{ cm} \\ &= \langle 1.90\cos 60^\circ - 2.80\cos 60^\circ, -1.90\sin 60^\circ - 2.80\sin 60^\circ\rangle \text{ cm} \\ &\approx \langle -0.45, -4.07\rangle \text{ cm} \end{aligned}$$



The magnitudes of these vectors are

$$\begin{aligned}|\mathbf{A}+\mathbf{B}| &= \sqrt{(2.80\cos 60^\circ + 1.90\cos 60^\circ \text{ cm})^2 + (2.80\sin 60^\circ - 1.90\sin 60^\circ \text{ cm})^2} \\ ] &\approx \sqrt{(2.35 \text{ cm})^2 + (0.779 \text{ cm})^2} \\ &\approx 2.48 \text{ cm}\end{aligned}$$

$$\begin{aligned}|\mathbf{A}-\mathbf{B}| &= \sqrt{(2.80\cos 60^\circ - 1.90\cos 60^\circ \text{ cm})^2 + (2.80\sin 60^\circ + 1.90\sin 60^\circ \text{ cm})^2} \\ &\approx \sqrt{(0.45 \text{ cm})^2 + (4.07 \text{ cm})^2} \\ &\approx 4.09 \text{ cm}\end{aligned}$$

$$\begin{aligned}|\mathbf{B}-\mathbf{A}| &= \sqrt{(1.90\cos 60^\circ \text{ cm} - 2.80\cos 60^\circ)^2 + (-1.90\sin 60^\circ \text{ cm} - 2.80\sin 60^\circ)^2} \\ &\approx \sqrt{(-0.45 \text{ cm})^2 + (-4.07 \text{ cm})^2} \\ &\approx 4.09 \text{ cm},\end{aligned}$$

and the angles measured counterclockwise from the positive x-axis are

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{2.80\sin 60^\circ - 1.90\sin 60^\circ \text{ cm}}{2.80\cos 60^\circ + 1.90\cos 60^\circ \text{ cm}}\right) \\ &\approx \tan^{-1}\left(\frac{0.779}{2.35}\right) \\ &\approx 18.3^\circ\end{aligned}$$

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{2.80\sin 60^\circ + 1.90\sin 60^\circ \text{ cm}}{2.80\cos 60^\circ - 1.90\cos 60^\circ \text{ cm}}\right) \\ &\approx \tan^{-1}\left(\frac{4.07}{0.45}\right) \\ &\approx 83.7^\circ\end{aligned}$$

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{-1.90\sin 60^\circ - 2.80\sin 60^\circ \text{ cm}}{1.90\cos 60^\circ - 2.80\cos 60^\circ \text{ cm}}\right) \\ &\approx \tan^{-1}\left(\frac{-4.07}{-0.45}\right) \\ &= \pi + \tan^{-1}\left(\frac{4.07}{0.45}\right) \\ &\approx 264^\circ,\end{aligned}$$

respectively.